LAMPIRAN 1

PROGRAM UNTUK MENGHITUNG INTEGRAL PROSES *RENEWAL* SECARA NUMERIK

Tampilan depan program.

```
##=============================================##
### PLEASE ONLY EDIT IN HERE AND ON 'input_here.csv' FILE ###
N = 4000 #this is for iteration
col_name = ['Type of Distribution', 't', 'P_1', 'P_2', 'f_t_mean', 'f_t_var', 'error_mean', 'error_var']
##=============================================##
##=============================================##
## STARTING FROM HERE, DO NOT EDIT ##
##=============================================##
##=============================================##
import time

tic = time.clock()

### READ AN INPUT FROM CSV FILE FORMAT ###

# in here, the CSV file would be converted into a list.
# the numbers would be converted into float numbers while `t` and `l` variables stay.
```

```
# float numbers represent real numbers and are written with
↪ a decimal point dividing the integer and fractional
↪ parts.
# the header of the data and the 'distribution type' column
↪ still can be seen in here.
import read_csv

all_data = read_csv.read('input_input.csv')

print ('ALL_DATA')
print (all_data)
print ('\n')

##===============================================##
### type DATA ###
type_data = []
for i in range(1,len(all_data)):
    type_data.append(all_data[i][0])

print ('type_DATA')
print (type_data)
print ('\n')

### t VARIABLE DATA ###
t_data = []
for i in range(1,len(all_data)):
    t_data.append(all_data[i][1])
```

```
print ('t_DATA')
print (t_data)
print ('\n')

### P_1 VARIABLE DATA ###
P_1_data = []
for i in range(1,len(all_data)):
    P_1_data.append(all_data[i][2])

print ('P_1_DATA')
print (P_1_data)
print ('\n')

### P_2 VARIABLE DATA ###
P_2_data = []
for i in range(1,len(all_data)):
    P_2_data.append(all_data[i][3])

print ('P_2_DATA')
print (P_2_data)
print ('\n')

##==============================================##
### PDF OF SOME CONTINUOUS DISTRIBUTION ###

### GAMMA DISTRIBUTION ###
```

```
# Gamma distribution defined by two parameters, beta and alpha
# beta  is inverse scale parameter of theta or rate parameter
# and alpha is shape parameter
# both parameters are positive real numbers.
# only alpha parameter that we will use here
# example, if there are Gamma(1,2) and Gamma(0.5,2), where 1 and 0.5 is beta and 2 is gamma
# then this (section of) program will only read it as Gamma(beta,2)
# where beta will be inputted later along with inverting
# pdf_data is a list contains results of gamma distribution calculation.

### UNIFORM DISTRIBUTION ###
# Uniform distribution is a bounded continuous distribution on [a, b]
# defined by two parameters a and b, which are it's min and max values.

### EXPONENTIAL DISTRIBUTION ###
# Exponential distribution defined by one parameter lambda, which is a rate.
import gamma_pdf
import uniform_pdf
import exponential_pdf
```

```python

pdf_data = []
for i in range(1, len(all_data)):
    if all_data[i][0].lower() == 'gamma':
        pdf_data.append(gamma_pdf.g_pdf(all_data[i][2], all_data[i][3]))
    elif all_data[i][0].lower() == 'uniform':
        pdf_data.append(uniform_pdf.u_pdf(all_data[i][2], all_data[i][3]))
    elif all_data[i][0].lower() == 'exponential':
        pdf_data.append(exponential_pdf.exp_pdf(all_data[i][2]))
    else:
        pdf_data.append(0)

print ('PDF_DATA')
print (pdf_data)
print ('\n')

##==============================================##
### LAPLACE TRANSFORM ###
# Laplace transform (t->s)
# -- "This function returns (F, a, cond) where F is the Laplace transform of f,
# Re(s)>a is the half-plane of convergence, and cond are auxiliary convergence conditions." --
# That means the output would be a tuple
```

```
# While we only need F for next steps
import laplace_transformation

transform_results = []
transform_data = []
for i in range(len(pdf_data)):
    transform_results.append(laplace_transformation.laplace_
    ↪ trans(pdf_data[i]))

# To eliminate unnecessary data and only left with F
for j in range(len(pdf_data)):
    transform_data.append(transform_results[j][0])

print ('TRANSFORM_DATA')
print (transform_data)
print ('\n')

##==============================================##
### SUBSTITUTE LAPLACE TRANSFORM OF GAMMA DISTRIBUTION TO
↪ EQUATION 1 (FOR MEAN), (SUYONO,2002:57) ###
import mean_eq

for_mean_data = []
for i in range(len(transform_data)):
    for_mean_data.append(mean_eq.subst(transform_data[i]))

print ('FOR_MEAN_DATA')
```

```
print (for_mean_data)
print ('\n')

##============================================##
### SUBSTITUTE LAPLACE TRANSFORM OF GAMMA DISTRIBUTION TO EQUATION 2 (FOR VARIANCE), (SUYONO,2002:57) ###
import var_eq

for_var_data = []
for i in range(len(transform_data)):
    for_var_data.append(var_eq.subst(transform_data[i]))

print ('FOR_VAR_DATA')
print (for_var_data)
print ('\n')

##============================================##
### Inverse Laplace Transforms Numerically ###
# inverse_numerically has three parameters: f, t, and K
# f is a data of density function that has been transformed and substituted to EQ 1 (for mean) or EQ 2 (for variance)
# t is a time, inputted automatically from "t_data" list
# K is a number of iteration, can be inputted manually and randomly by changing the value of N in line 3 on this script
import inverse_laplace_trans_numerically
```

```python

# FOR MEAN
invtrans_num_mean = []
for i in range(len(transform_data)):
    invtrans_num_mean.append(inverse_laplace_trans_numerically.inverse_numerically(for_mean_data[i], t_data[i], N))

# FOR VAR
invtrans_num_var = []
for i in range(len(transform_data)):
    invtrans_num_var.append(inverse_laplace_trans_numerically.inverse_numerically(for_var_data[i], t_data[i], N))

print ('INV_TRANS_NUM_MEAN')
print (invtrans_num_mean)
print ('\n')
print ('INV_TRANS_NUM_VAR')
print (invtrans_num_var)
print ('\n')

# ERROR FOR MEAN
import error
e_m = []
for i in range(len(transform_data)):
    e_m.append(error.error_n(pdf_data[i], t_data[i], N))
```

```

# ERROR FOR VARIANCE
e_v = []
for i in range(len(transform_data)):
    e_v.append(error.error_n(pdf_data[i], t_data[i], N))

print ('ERROR_MEAN')
print (e_m)
print ('\n')
print ('ERROR_VAR')
print (e_v)
print ('\n')

##=============================================##
### FINAL RESULTS ###
# f(t) = numerically - error

# MEAN
f_t_mean = []
for i in range(len(transform_data)):
    f_t_mean.append(invtrans_num_mean[i] - e_m[i])

# VARIANCE
f_t_var = []
for i in range(len(transform_data)):
    f_t_var.append(invtrans_num_var[i] - e_v[i])

```

```
print ('f_t_MEAN')
print (f_t_mean)
print ('\n')
print ('f_t_VAR')
print (f_t_var)
print ('\n')

##==============================================##
### TIMING THE COMPUTING TIME ###
#import computing_time
#print(computing_time.processing_time())
##==============================================##
### SAVE OUTPUT AS CSV ###
import save_as_new_csv

save_as_new_csv.write(col_name, [type_data, t_data,
↪ P_1_data, P_2_data, f_t_mean, f_t_var, e_m, e_v],
↪ 'output.csv')

toc = time.clock()
time_sec = toc - tic
print ('PROCESSING TIME IN MINUTES')
print (time_sec / 60)
print ('PROCESSING TIME IN HOURS')
print (time_sec / 3600)
```

Program untuk membaca data dari file input.csv.

```
##read CSV
def read(input):
    f = open(input, "r")
    lines = f.read().split("\n") # "\r\n" if needed

    arr_str = []
    ##make data into a list
    for line in lines:
        if line != "": # add other needed checks to skip titles
            arr_str.append(line.split(","))

    for i in xrange(1, len(arr_str)):
        for j in xrange(1, len(arr_str[i])):
            if arr_str[i][j] == '-':
                arr_str[i][j] = 0
            else:
                arr_str[i][j] = float(arr_str[i][j])

    return arr_str
```

Program untuk menghitung f.k.p dari distribusi Gamma.

```
from scipy.special import gamma
from sympy import exp
from sympy.abc import x

def g_pdf(b, a):
```

```
    pdf_result = b ** a * x ** (a - 1) * exp(-b * x) /
    ↪ gamma(a)
    return pdf_result
```

Program untuk menghitung f.k.p dari distribusi Eksponensial.

```
from sympy import exp
from sympy.abc import x

def exp_pdf(l):
    pdf_result = l * exp(-2 * x)
    return pdf_result
```

Program untuk menghitung f.k.p dari distribusi Uniform.

```
from sympy import sympify

def u_pdf(a,b):
    pdf_result = sympify(1 / (b - a))
    return pdf_result
```

Program untuk melakukan transformasi terhadap distribusi Gamma, Eksponensial, dan Uniform menggunakan transformasi Laplace.

```
from sympy.abc import x, s, t
from sympy.integrals import laplace_transform

# Laplace transform (t->s)
def laplace_trans(f_s):
    #T = laplace_transform(f_s.subs({x: t}), t, s)
    if x in f_s.free_symbols == False:
```

```
        T = laplace_transform(f_s, t, s)
    else:
        T = laplace_transform(f_s.subs({x:t}), t, s)
    return T
```

Program untuk mensubstitusi hasil transformasi Laplace ke Persamaan (2.22) (untuk mean).

```
from sympy import *
from sympy.abc import s, t, l

def subst (f_s):
    subs_mean_trans = f_s / (s**2 * (1 - f_s))
    return subs_mean_trans
    #simplified_sub_mean_trans = simplify(subs_mean_trans)
    #return simplified_sub_mean_trans
```

Program untuk mensubstitusi hasil transformasi Laplace ke Persamaan (2.23) (untuk variansi).

```
from sympy import *
from sympy import integrate, exp, oo
from sympy.abc import s, t, l

def subst (f_s):
    subs_var_trans = (2 * f_s) * (1 - (f_s)**2 + (s
    ↪ *integrate(f_s * (t*exp(-s*t)),(t,0,
    ↪ oo),conds='none'))) / (s**3 * ((1 - f_s)**3))
    simplified_sub_var_trans = simplify(subs_var_trans)
    return simplified_sub_var_trans
```

```

# ` conds='none' ` is used in integrate to access parts of an expression only
```

Program untuk mencari nilai $f(t)$ (sebelum dikurangi dengan error) untuk mean dan variansi.

```
from sympy import *
from sympy import I, exp
from sympy.abc import s, t, l

def inverse_numerically(f_s, t, K):
    A = 7
    P = exp(A).evalf() / (2 * t)
    B_1 = A / t
    #B_2_im = complex(0, math.pi / t)
    B_2_im = pi.evalf()*I / t
    f_1 = f_s.subs({s:B_1})
    f_t_1 = P * f_1

    f_t2 = []
    for i in xrange(1, K+1):
        f_t2.append((-1) ** i * re(complex(simplify(f_s.subs({s: B_1 + (B_2_im * i)})))))

    r = sum(f_t2)
    f_t_2 = 2 * P * re(r)
    f_t_3 = f_t_1 + f_t_2
```

```
    return f_t_3
```

Program untuk mencari nilai error untuk mean dan variansi.

```
import math
from sympy import *
from sympy.abc import s, l, x

def error_n(f_t, t, K):
    A = 4
    e_r = []
    for j in xrange(1, K+1):
        e_r.append(math.exp(-2 * j * A) *
         ↪  simplify((f_t.subs({s: (2 * j * t) + t, x:t}))))

    e = float(sum(e_r))
    return e
```

Hasil dari program.

```
C:\ProgramData\Anaconda2\python.exe D:/1/home.py
ALL_DATA
[['Type of Distribution', 't', 'P_1', 'P_2'], ['Gamma', 2.0,
 ↪  1.0, 0.5], ['Exponential', 2.0, 2.0, 0], ['Uniform',
 ↪  2.0, 0.0, 1.0]]


type_DATA
['Gamma', 'Exponential', 'Uniform']

```

```

t_DATA
[2.0, 2.0, 2.0]


P_1_DATA
[1.0, 2.0, 0.0]


P_2_DATA
[0.5, 0, 1.0]


PDF_DATA
[0.564189583547756*x**(-0.5)*exp(-1.0*x), 2.0*exp(-2*x), 1.00000000000000]


TRANSFORM_DATA
[1.0*(1.0*s + 1)**(-0.5), 2.0/(s + 2), 1.0/s]


FOR_MEAN_DATA
[1.0*(1.0*s + 1)**(-0.5)/(s**2*(-1.0*(1.0*s + 1)**(-0.5) + 1)), 2.0/(s**2*(1 - 2.0/(s + 2))*(s + 2)), 1.0/(s**3*(1 - 1.0/s))]

```

```

FOR_VAR_DATA
[2.0*(1.0*s + 1)**(-0.5)*(s*(1.0*s + 1)**0.5 - s*(1.0*s +
↪  1)**1.5 - (1.0*s + 1)**1.0)/(s**4*(-(1.0*s + 1)**0.5 +
↪  1.0)**3), (4.0*s*(s + 2)**2 - 8.0*s + 16.0)/s**7,
↪  2.0/(s*(s - 1.0)**3)]


INV_TRANS_NUM_MEAN
[4.87919642134218, 4.00002993509933, 4.38938575607892]


INV_TRANS_NUM_VAR
[36.1797620093537, 19.5576869550147, 12.7868338121042]


ERROR_MEAN
[1.8118029431276015e-05, 1.2292548397245378e-05,
↪  0.00033557520084124496]


ERROR_VAR
[1.8118029431276015e-05, 1.2292548397245378e-05,
↪  0.00033557520084124496]


f_t_MEAN
```

```
[4.87917830331275, 4.00001764255094, 4.38905018087808]


f_t_VAR
[36.1797438913243, 19.5576746624663, 12.7864982369033]


PROCESSING TIME IN MINUTES
162.871024538
PROCESSING TIME IN HOURS
2.71451707564

Process finished with exit code 0
```

Output ataupun hasil dari 4.2 ditulis pada *file* output.csv secara otomatis. Berikut program untuk menulis *output*.

```
import csv
from itertools import izip_longest

def write(col_name, output_list, output_csv_file_name):
    export_data = izip_longest(*output_list)
    with open(output_csv_file_name, 'wb') as writeFile:
        wr = csv.writer(writeFile)
        wr.writerow((col_name))
        wr.writerows((export_data))
    writeFile.close()
```

LAMPIRAN 2

TABEL PERHITUNGAN *MEAN* TOTAL WAKTU TUNGGU UNTUK BEBERAPA KASUS

Tabel-tabel dibawah ini digunakan untuk melihat kestabilan dari banyaknya iterasi yang diperlukan, N, dan bilangan riil yang dipilih, A, yang sesuai. Beberapa kasus yang dapat dicari nilai analitiknya digunakan untuk melihat apakah hasil perhitungan secara numerik sudah mendekati nilai perhitungan secara analitik atau belum. Dari 40 kasus dibawah ini, diambil kesimpulan bahwa dengan N = 4000, nilainya sudah stabil sampai tujuh angka di belakang koma. Lalu dari beberapa kasus yang dapat dicari nilai analitiknya, ketika nilai A = 7, nilai perhitungan secara numerik memiliki nilai yang mendekati nilai analitik sampai empat atau lima angka di belakang koma. Karenanya pada program di Lampiran 1, digunakan nilai N = 4000 dan A = 7.

Berikut adalah beberapa kasus yang dapat dicari nilainya secara analitik.

Gamma (1, 1), t = 1, mean secara analitik: 0.5

N	**A = 4**	**A = 5**	**A = 6**	**A = 7**
1000	0.5013875	0.5000809	0.4999042	0.4998803
2000	0.5013875	0.5000809	0.4999042	0.4998803
3000	0.5013875	0.5000809	0.4999042	0.4998803
4000	0.5013875	0.5000809	0.4999042	0.4998803
5000	0.5013875	0.5000809	0.4999042	0.4998803
6000	0.5013875	0.5000809	0.4999042	0.4998803

Tabel 4.1: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 1) dan t = 1

Gamma (1, 1), t = 2, mean secara analitik: 2

N	A = 4	A = 5	A = 6	A = 7
1000	2.0059985	2.0007719	2.0000652	1.9999696
2000	2.0059985	2.0007719	2.0000652	1.9999696
3000	2.0059985	2.0007719	2.0000652	1.9999696
4000	2.0059985	2.0007719	2.0000652	1.9999696
5000	2.0059985	2.0007719	2.0000652	1.9999696
6000	2.0059985	2.0007719	2.0000652	1.9999696

Tabel 4.2: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 1) dan t = 2

Gamma (1, 1), t = 3, mean secara analitik: 4.5

N	A = 4	A = 5	A = 6	A = 7
1000	4.5135822	4.5018222	4.5002321	4.500017
2000	4.5135822	4.5018222	4.5002321	4.500017
3000	4.5135822	4.5018222	4.5002321	4.500017
4000	4.5135822	4.5018222	4.5002321	4.500017
5000	4.5135822	4.5018222	4.5002321	4.500017
6000	4.5135822	4.5018222	4.5002321	4.500017

Tabel 4.3: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 1) dan t = 3

Gamma (1, 1), t = 4, mean secara analitik: 8

N	A = 4	A = 5	A = 6	A = 7
1000	8.0241697	8.0032631	8.0004362	8.0000537
2000	8.0241697	8.0032631	8.0004362	8.0000537
3000	8.0241697	8.0032631	8.0004362	8.0000537
4000	8.0241697	8.0032631	8.0004362	8.0000537
5000	8.0241697	8.0032631	8.0004362	8.0000537
6000	8.0241697	8.0032631	8.0004362	8.0000537

Tabel 4.4: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 1) dan t = 4

Gamma (1, 1), t = 5, mean secara analitik: 12.5

N	A = 4	A = 5	A = 6	A = 7
1000	12.5377725	12.5051059	12.500689	12.5000913
2000	12.5377725	12.5051059	12.500689	12.5000913
3000	12.5377725	12.5051059	12.500689	12.5000913
4000	12.5377725	12.5051059	12.500689	12.5000913
5000	12.5377725	12.5051059	12.500689	12.5000913
6000	12.5377725	12.5051059	12.500689	12.5000913

Tabel 4.5: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 1) dan t = 5

Gamma (1, 2), t = 1, mean secara analitik: 0.1080830896

N	A = 4	A = 5	A = 6	A = 7
1000	0.1085052	0.1080334	0.1079696	0.107961
2000	0.1085052	0.1080334	0.1079696	0.107961
3000	0.1085052	0.1080334	0.1079696	0.107961
4000	0.1085052	0.1080334	0.1079696	0.107961
5000	0.1085052	0.1080334	0.1079696	0.107961
6000	0.1085052	0.1080334	0.1079696	0.107961

Tabel 4.6: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 2) dan t = 1

Gamma (1, 2), t = 2, mean secara analitik: 0.6227105451

N	A = 4	A = 5	A = 6	A = 7
1000	0.6251802	0.6229659	0.6226666	0.6226261
2000	0.6251802	0.6229659	0.6226666	0.6226261
3000	0.6251802	0.6229659	0.6226666	0.6226261
4000	0.6251802	0.6229659	0.6226666	0.6226261
5000	0.6251802	0.6229659	0.6226666	0.6226261
6000	0.6251802	0.6229659	0.6226666	0.6226261

Tabel 4.7: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 2) dan t = 2

Gamma (1, 2), t = 3, mean secara analitik: 1.624690156

N	A = 4	A = 5	A = 6	A = 7
1000	1.6307262	1.625463	1.6247514	1.6246551
2000	1.6307262	1.625463	1.6247514	1.6246551
3000	1.6307262	1.625463	1.6247514	1.6246551
4000	1.6307262	1.625463	1.6247514	1.6246551
5000	1.6307262	1.625463	1.6247514	1.6246551
6000	1.6307262	1.625463	1.6247514	1.6246551

Tabel 4.8: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 2) dan t = 1

Gamma (1, 2), t = 4, mean secara analitik: 3.124958067

N	A = 4	A = 5	A = 6	A = 7
1000	3.1360564	3.1264376	3.125137	3.124961
2000	3.1360564	3.1264376	3.125137	3.124961
3000	3.1360564	3.1264376	3.125137	3.124961
4000	3.1360564	3.1264376	3.125137	3.124961
5000	3.1360564	3.1264376	3.125137	3.124961
6000	3.1360564	3.1264376	3.125137	3.124961

Tabel 4.9: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 2) dan t = 4

Gamma (1, 2), t = 5, mean secara analitik: 5.124994325

N	A = 4	A = 5	A = 6	A = 7
1000	5.1426536	5.1273725	5.1253064	5.1250268
2000	5.1426536	5.1273725	5.1253064	5.1250268
3000	5.1426536	5.1273725	5.1253064	5.1250268
4000	5.1426536	5.1273725	5.1253064	5.1250268
5000	5.1426536	5.1273725	5.1253064	5.1250268
6000	5.1426536	5.1273725	5.1253064	5.1250268

Tabel 4.10: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 2) dan t = 5

Gamma (0.5, 1), t = 1, mean secara analitik: 0.25

N	A = 4	A = 5	A = 6	A = 7
1000	0.2506537	0.2500004	0.2499121	0.2499001
2000	0.2506537	0.2500004	0.2499121	0.2499001
3000	0.2506537	0.2500004	0.2499121	0.2499001
4000	0.2506537	0.2500004	0.2499121	0.2499001
5000	0.2506537	0.2500004	0.2499121	0.2499001
6000	0.2506537	0.2500004	0.2499121	0.2499001

Tabel 4.11: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 1) dan t = 1

Gamma (0.5, 1), t = 2, mean secara analitik: 1

N	A = 4	A = 5	A = 6	A = 7
1000	1.0029603	1.0003469	0.9999936	0.9999458
2000	1.0029603	1.0003469	0.9999936	0.9999458
3000	1.0029603	1.0003469	0.9999936	0.9999458
4000	1.0029603	1.0003469	0.9999936	0.9999458
5000	1.0029603	1.0003469	0.9999936	0.9999458
6000	1.0029603	1.0003469	0.9999936	0.9999458

Tabel 4.12: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 1) dan t = 2

Gamma (0.5, 1), t = 3, mean secara analitik: 2.25

N	A = 4	A = 5	A = 6	A = 7
1000	2.256762	2.250882	2.250087	2.2499794
2000	2.256762	2.250882	2.250087	2.2499794
3000	2.256762	2.250882	2.250087	2.2499794
4000	2.256762	2.250882	2.250087	2.2499794
5000	2.256762	2.250882	2.250087	2.2499794
6000	2.256762	2.250882	2.250087	2.2499794

Tabel 4.13: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 1) dan t = 3

Gamma (0.5, 1), t = 4, mean secara analitik: 4

N	A = 4	A = 5	A = 6	A = 7
1000	4.0120652	4.0016119	4.0001985	4.0000072
2000	4.0120652	4.0016119	4.0001985	4.0000072
3000	4.0120652	4.0016119	4.0001985	4.0000072
4000	4.0120652	4.0016119	4.0001985	4.0000072
5000	4.0120652	4.0016119	4.0001985	4.0000072
6000	4.0120652	4.0016119	4.0001985	4.0000072

Tabel 4.14: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 1) dan t = 4

Gamma (0.5, 1), t = 5, mean secara analitik: 6.25

N	A = 4	A = 5	A = 6	A = 7
1000	6.2688736	6.2525403	6.2503318	6.250033
2000	6.2688736	6.2525403	6.2503318	6.250033
3000	6.2688736	6.2525403	6.2503318	6.250033
4000	6.2688736	6.2525403	6.2503318	6.250033
5000	6.2688736	6.2525403	6.2503318	6.250033
6000	6.2688736	6.2525403	6.2503318	6.250033

Tabel 4.15: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 1) dan t = 5

Gamma (0.5, 2), t = 1, mean secara analitik: 0.0330301397

N	A = 4	A = 5	A = 6	A = 7
1000	0.033185	0.0330071	0.032983	0.0329798
2000	0.033185	0.0330071	0.032983	0.0329798
3000	0.033185	0.0330071	0.032983	0.0329798
4000	0.033185	0.0330071	0.032983	0.0329798
5000	0.033185	0.0330071	0.032983	0.0329798
6000	0.033185	0.0330071	0.032983	0.0329798

Tabel 4.16: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 2) dan t = 1

Gamma (0.5, 2), t = 2, mean secara analitik: 0.2161661792

N	A = 4	A = 5	A = 6	A = 7
1000	0.2171957	0.216252	0.2161244	0.2161072
2000	0.2171957	0.216252	0.2161244	0.2161072
3000	0.2171957	0.216252	0.2161244	0.2161072
4000	0.2171957	0.216252	0.2161244	0.2161072
5000	0.2171957	0.216252	0.2161244	0.2161072
6000	0.2171957	0.216252	0.2161244	0.2161072

Tabel 4.17: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 2) dan t = 2

Gamma (0.5, 2), t = 3, mean secara analitik: 0.6125532329

N	A = 4	A = 5	A = 6	A = 7
1000	0.6152255	0.612866	0.612547	0.6125038
2000	0.6152255	0.612866	0.612547	0.6125038
3000	0.6152255	0.612866	0.612547	0.6125038
4000	0.6152255	0.612866	0.612547	0.6125038
5000	0.6152255	0.612866	0.612547	0.6125038
6000	0.6152255	0.612866	0.612547	0.6125038

Tabel 4.18: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 2) dan t = 3

Gamma (0.5, 2), t = 4, mean secara analitik: 1.24542109

N	A = 4	A = 5	A = 6	A = 7
1000	1.2504966	1.2460681	1.2454694	1.2453884
2000	1.2504966	1.2460681	1.2454694	1.2453884
3000	1.2504966	1.2460681	1.2454694	1.2453884
4000	1.2504966	1.2460681	1.2454694	1.2453884
5000	1.2504966	1.2460681	1.2454694	1.2453884
6000	1.2504966	1.2460681	1.2454694	1.2453884

Tabel 4.19: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 2) dan t = 4

Gamma (0.5, 2), t = 5, mean secara analitik: 2.123315513

N	A = 4	A = 5	A = 6	A = 7
1000	2.13155	2.1243992	2.1234324	2.1233016
2000	2.13155	2.1243992	2.1234324	2.1233016
3000	2.13155	2.1243992	2.1234324	2.1233016
4000	2.13155	2.1243992	2.1234324	2.1233016
5000	2.13155	2.1243992	2.1234324	2.1233016
6000	2.13155	2.1243992	2.1234324	2.1233016

Tabel 4.20: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 2) dan t = 5

Berikut adalah beberapa kasus yang tidak dapat dicari nilainya secara analitik.

Gamma (1, 0.5), t = 1, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	1.3964427	1.3934299	1.3930224	1.3929668
2000	1.3964427	1.39343	1.3930226	1.3929674
3000	1.3964427	1.39343	1.3930226	1.3929674
4000	1.3964427	1.39343	1.3930226	1.3929675
5000	1.3964427	1.39343	1.3930226	1.3929675
6000	1.3964427	1.39343	1.3930226	1.3929675

Tabel 4.21: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 0.5) dan t = 1

Gamma (1, 0.5), t = 2, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	4.8921989	4.8809109	4.8793842	4.8791764
2000	4.892199	4.8809111	4.8793848	4.879178
3000	4.892199	4.8809112	4.8793849	4.8791782
4000	4.892199	4.8809112	4.8793849	4.8791783
5000	4.892199	4.8809112	4.8793849	4.8791783
6000	4.892199	4.8809112	4.8793849	4.8791783

Tabel 4.22: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 0.5) dan t = 2

Gamma (1, 0.5), t = 3, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	10.4047549	10.3799648	10.3766121	10.3761562
2000	10.4047551	10.3799652	10.3766132	10.3761592
3000	10.4047551	10.3799653	10.3766133	10.3761596
4000	10.4047551	10.3799653	10.3766134	10.3761597
5000	10.4047551	10.3799653	10.3766134	10.3761598
6000	10.4047551	10.3799653	10.3766134	10.3761598

Tabel 4.23: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 0.5) dan t = 3

Gamma (1, 0.5), t = 4, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	17.9256297	17.8821107	17.8762252	17.8754253
2000	17.9256299	17.8821114	17.8762269	17.87543
3000	17.9256299	17.8821115	17.8762271	17.8754306
4000	17.92563	17.8821115	17.8762272	17.8754308
5000	17.92563	17.8821115	17.8762272	17.8754309
6000	17.92563	17.8821115	17.8762272	17.8754309

Tabel 4.24: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 0.5) dan t = 4

Gamma (1, 0.5), t = 5, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	27.4531146	27.3856401	27.376515	27.3752754
2000	27.4531149	27.385641	27.3765174	27.3752819
3000	27.453115	27.3856411	27.3765177	27.3752828
4000	27.453115	27.3856412	27.3765178	27.3752831
5000	27.453115	27.3856412	27.3765178	27.3752832
6000	27.453115	27.3856412	27.3765178	27.3752832

Tabel 4.25: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 0.5) dan t = 5

Gamma (1, 2.5), t = 1, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	0.051417	0.0511046	0.0510624	0.0510567
2000	0.051417	0.0511046	0.0510624	0.0510567
3000	0.051417	0.0511046	0.0510624	0.0510567
4000	0.051417	0.0511046	0.0510624	0.0510567
5000	0.051417	0.0511046	0.0510624	0.0510567
6000	0.051417	0.0511046	0.0510624	0.0510567

Tabel 4.26: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 2.5) dan t = 1

Gamma (1, 2.5), t = 2, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	0.3755324	0.3739134	0.3736945	0.3736649
2000	0.3755324	0.3739134	0.3736945	0.3736649
3000	0.3755324	0.3739134	0.3736945	0.3736649
4000	0.3755324	0.3739134	0.3736945	0.3736649
5000	0.3755324	0.3739134	0.3736945	0.3736649
6000	0.3755324	0.3739134	0.3736945	0.3736649

Tabel 4.27: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 2.5) dan t = 2

Gamma (1, 2.5), t = 3, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	1.0799067	1.0759356	1.0753987	1.0753261
2000	1.0799067	1.0759356	1.0753987	1.0753261
3000	1.0799067	1.0759356	1.0753987	1.0753261
4000	1.0799067	1.0759356	1.0753987	1.0753261
5000	1.0799067	1.0759356	1.0753987	1.0753261
6000	1.0799067	1.0759356	1.0753987	1.0753261

Tabel 4.28: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 2.5) dan t = 3

Gamma (1, 2.5), t = 4, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	2.1836352	2.1762667	2.1752704	2.1751356
2000	2.1836352	2.1762667	2.1752704	2.1751356
3000	2.1836352	2.1762667	2.1752704	2.1751356
4000	2.1836352	2.1762667	2.1752704	2.1751356
5000	2.1836352	2.1762667	2.1752704	2.1751356
6000	2.1836352	2.1762667	2.1752704	2.1751356

Tabel 4.29: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 2.5) dan t = 4

Gamma (1, 2.5), t = 5, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	3.6886733	3.676862	3.6752651	3.675049
2000	3.6886733	3.676862	3.6752651	3.675049
3000	3.6886733	3.676862	3.6752651	3.675049
4000	3.6886733	3.676862	3.6752651	3.675049
5000	3.6886733	3.676862	3.6752651	3.675049
6000	3.6886733	3.676862	3.6752651	3.675049

Tabel 4.30: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (1, 2.5) dan t = 5

Gamma (0.5, 0.5), t = 1, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	0.8351705	0.8334961	0.8332695	0.8332386
2000	0.8351705	0.8334961	0.8332697	0.833239
3000	0.8351705	0.8334961	0.8332697	0.833239
4000	0.8351705	0.8334961	0.8332697	0.8332391
5000	0.8351705	0.8334961	0.8332697	0.8332391
6000	0.8351705	0.8334961	0.8332697	0.8332391

Tabel 4.31: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 0.5) dan t = 1

Gamma (0.5, 0.5), t = 2, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	2.7929899	2.7869643	2.7861492	2.786038
2000	2.7929899	2.7869644	2.7861496	2.7860392
3000	2.7929899	2.7869644	2.7861497	2.7860394
4000	2.7929899	2.7869644	2.7861497	2.7860394
5000	2.7929899	2.7869644	2.7861497	2.7860394
6000	2.7929899	2.7869644	2.7861497	2.7860394

Tabel 4.32: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 0.5) dan t = 2

Gamma (0.5, 0.5), t = 3, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	5.781943	5.7689492	5.7671916	5.7669522
2000	5.7819431	5.7689495	5.7671924	5.7669544
3000	5.7819431	5.7689495	5.7671925	5.7669547
4000	5.7819431	5.7689495	5.7671925	5.7669547
5000	5.7819432	5.7689495	5.7671925	5.7669548
6000	5.7819432	5.7689495	5.7671926	5.7669548

Tabel 4.33: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 0.5) dan t = 3

Gamma (0.5, 0.5), t = 4, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	9.7844249	9.761849	9.7587955	9.7583799
2000	9.7844251	9.7618494	9.7587967	9.7583832
3000	9.7844251	9.7618495	9.7587969	9.7583837
4000	9.7844251	9.7618495	9.7587969	9.7583838
5000	9.7844251	9.7618495	9.758797	9.7583838
6000	9.7844251	9.7618495	9.758797	9.7583839

Tabel 4.34: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 0.5) dan t = 4

Gamma (0.5, 0.5), t = 5, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	14.7944256	14.7596542	14.7549514	14.7543116
2000	14.7944258	14.7596548	14.754953	14.7543162
3000	14.7944258	14.7596549	14.7549533	14.7543168
4000	14.7944258	14.7596549	14.7549533	14.754317
5000	14.7944258	14.7596549	14.7549534	14.7543171
6000	14.7944258	14.7596549	14.7549534	14.7543171

Tabel 4.35: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 0.5) dan t = 5

Gamma (0.5, 2.5), t = 1, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	0.0116933	0.0115956	0.0115824	0.0115806
2000	0.0116933	0.0115956	0.0115824	0.0115806
3000	0.0116933	0.0115956	0.0115824	0.0115806
4000	0.0116933	0.0115956	0.0115824	0.0115806
5000	0.0116933	0.0115956	0.0115824	0.0115806
6000	0.0116933	0.0115956	0.0115824	0.0115806

Tabel 4.36: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 2.5) dan t = 1

Gamma (0.5, 2.5), t = 2, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	0.1029732	0.1023485	0.1022641	0.1022527
2000	0.1029732	0.1023485	0.1022641	0.1022527
3000	0.1029732	0.1023485	0.1022641	0.1022527
4000	0.1029732	0.1023485	0.1022641	0.1022527
5000	0.1029732	0.1023485	0.1022641	0.1022527
6000	0.1029732	0.1023485	0.1022641	0.1022527

Tabel 4.37: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 2.5) dan t = 2

Gamma (0.5, 2.5), t = 3, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	0.3382663	0.3365963	0.3363706	0.33634
2000	0.3382663	0.3365963	0.3363706	0.33634
3000	0.3382663	0.3365963	0.3363706	0.33634
4000	0.3382663	0.3365963	0.3363706	0.33634
5000	0.3382663	0.3365963	0.3363706	0.33634
6000	0.3382663	0.3365963	0.3363706	0.33634

Tabel 4.38: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 2.5) dan t = 3

Gamma (0.5, 2.5), t = 4, mean secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	0.7512098	0.7479718	0.747534	0.7474748
2000	0.7512098	0.7479718	0.747534	0.7474748
3000	0.7512098	0.7479718	0.747534	0.7474748
4000	0.7512098	0.7479718	0.747534	0.7474748
5000	0.7512098	0.7479718	0.747534	0.7474748
6000	0.7512098	0.7479718	0.747534	0.7474748

Tabel 4.39: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 2.5) dan t = 4

Gamma (0.5, 2.5), t = 5, mean secara analitik: -

N	**A = 4**	**A = 5**	**A = 6**	**A = 7**
1000	1.3565229	1.3511942	1.3504737	1.3503762
2000	1.3565229	1.3511942	1.3504737	1.3503762
3000	1.3565229	1.3511942	1.3504737	1.3503762
4000	1.3565229	1.3511942	1.3504737	1.3503762
5000	1.3565229	1.3511942	1.3504737	1.3503762
6000	1.3565229	1.3511942	1.3504737	1.3503762

Tabel 4.40: Tabel Hasil Perhitungan Mean Secara Numerik Untuk Gamma (0.5, 2.5) dan t = 5

LAMPIRAN 3

TABEL PERHITUNGAN VARIANSI TOTAL WAKTU TUNGGU UNTUK BEBERAPA KASUS

Tabel-tabel dibawah ini digunakan untuk melihat kestabilan dari banyaknya iterasi yang diperlukan, N, dan bilangan riil yang dipilih, A, yang sesuai. Beberapa kasus yang dapat dicari nilai analitiknya digunakan untuk melihat apakah hasil perhitungan secara numerik sudah mendekati nilai perhitungan secara analitik atau belum. Dari 40 kasus dibawah ini, diambil kesimpulan bahwa dengan N = 4000, nilainya sudah stabil sampai tujuh angka di belakang koma. Lalu dari beberapa kasus yang dapat dicari nilai analitiknya, ketika nilai A = 7, nilai perhitungan secara numerik memiliki nilai yang mendekati nilai analitik sampai empat atau lima angka di belakang koma. Karenanya pada program di Lampiran 1, digunakan nilai N = 4000 dan A = 7.

Berikut adalah beberapa kasus yang dapat dicari nilainya secara analitik.

Gamma (1, 1), t = 1, variansi secara analitik: 0.5194444444

N	**A = 4**	**A = 5**	**A = 6**	**A = 7**
1000	0.528934	0.5206188	0.5194966	0.5193448
2000	0.528934	0.5206188	0.5194966	0.5193448
3000	0.528934	0.5206188	0.5194966	0.5193448
4000	0.528934	0.5206188	0.5194966	0.5193448
5000	0.528934	0.5206188	0.5194966	0.5193448
6000	0.528934	0.5206188	0.5194966	0.5193448

Tabel 4.41: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 1) dan t = 1

Gamma (1, 1), t = 2, variansi secara analitik: 6.044444444

N	A = 4	A = 5	A = 6	A = 7
1000	6.2286908	6.0692552	6.0477614	6.044854
2000	6.2286908	6.0692552	6.0477614	6.044854
3000	6.2286908	6.0692552	6.0477614	6.044854
4000	6.2286908	6.0692552	6.0477614	6.044854
5000	6.2286908	6.0692552	6.0477614	6.044854
6000	6.2286908	6.0692552	6.0477614	6.044854

Tabel 4.42: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 1) dan t = 2

Gamma (1, 1), t = 3, variansi secara analitik: 28.575

N	A = 4	A = 5	A = 6	A = 7
1000	29.854767	28.7474722	28.5983141	28.5781405
2000	29.854767	28.7474722	28.5983141	28.5781405
3000	29.854767	28.7474722	28.5983141	28.5781405
4000	29.854767	28.7474722	28.5983141	28.5781405
5000	29.854767	28.7474722	28.5983141	28.5781405
6000	29.854767	28.7474722	28.5983141	28.5781405

Tabel 4.43: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 1) dan t = 3

Gamma (1, 1), t = 4, variansi secara analitik: 92.44444444

N	A = 4	A = 5	A = 6	A = 7
1000	98.0000902	93.1928756	92.5456654	92.4581368
2000	98.0000902	93.1928756	92.5456654	92.4581367
3000	98.0000902	93.1928756	92.5456654	92.4581367
4000	98.0000902	93.1928756	92.5456654	92.4581367
5000	98.0000902	93.1928756	92.5456654	92.4581367
6000	98.0000902	93.1928756	92.5456654	92.4581367

Tabel 4.44: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 1) dan t = 4

Gamma (1, 1), t = 5, variansi secara analitik: 241.3194444

N	A = 4	A = 5	A = 6	A = 7
1000	259.4900325	243.7665328	241.6504	241.3642284
2000	259.4900325	243.7665328	241.6504	241.3642284
3000	259.4900325	243.7665328	241.6504	241.3642284
4000	259.4900325	243.7665328	241.6504	241.3642284
5000	259.4900325	243.7665328	241.6504	241.3642284
6000	259.4900325	243.7665328	241.6504	241.3642284

Tabel 4.45: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 1) dan t = 5

Gamma (1, 2), t = 1, variansi secara analitik: 0.0621277715

N	A = 4	A = 5	A = 6	A = 7
1000	0.063425	0.0621961	0.0620303	0.0620078
2000	0.063425	0.0621961	0.0620303	0.0620078
3000	0.063425	0.0621961	0.0620303	0.0620078
4000	0.063425	0.0621961	0.0620303	0.0620078
5000	0.063425	0.0621961	0.0620303	0.0620078
6000	0.063425	0.0621961	0.0620303	0.0620078

Tabel 4.46: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 2) dan t = 1

Gamma (1, 2), t = 2, variansi secara analitik: 0.8687420389

N	A = 4	A = 5	A = 6	A = 7
1000	0.894467	0.872134	0.8691223	0.868715
2000	0.894467	0.872134	0.8691223	0.868715
3000	0.894467	0.872134	0.8691223	0.868715
4000	0.894467	0.872134	0.8691223	0.868715
5000	0.894467	0.872134	0.8691223	0.868715
6000	0.894467	0.872134	0.8691223	0.868715

Tabel 4.47: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 2) dan t = 2

Gamma (1, 2), t = 3, variansi secara analitik: 4.223218397

N	A = 4	A = 5	A = 6	A = 7
1000	4.3917197	4.2458915	4.2262419	4.2235842
2000	4.3917197	4.2458915	4.2262419	4.2235842
3000	4.3917197	4.2458915	4.2262419	4.2235842
4000	4.3917197	4.2458915	4.2262419	4.2235842
5000	4.3917197	4.2458915	4.2262419	4.2235842
6000	4.3917197	4.2458915	4.2262419	4.2235842

Tabel 4.48: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 2) dan t = 1

Gamma (1, 2), t = 4, variansi secara analitik: 13.53545464

N	A = 4	A = 5	A = 6	A = 7
1000	14.2380832	13.6301078	13.5482357	13.537163
2000	14.2380832	13.6301078	13.5482357	13.537163
3000	14.2380832	13.6301078	13.5482357	13.537163
4000	14.2380832	13.6301078	13.5482357	13.537163
5000	14.2380832	13.6301078	13.5482357	13.537163
6000	14.2380832	13.6301078	13.5482357	13.537163

Tabel 4.49: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 2) dan t = 4

Gamma (1, 2), t = 5, variansi secara analitik: 34.61279544

N	A = 4	A = 5	A = 6	A = 7
1000	36.8558366	34.9149037	34.6536452	34.6183136
2000	36.8558366	34.9149037	34.6536452	34.6183136
3000	36.8558366	34.9149037	34.6536452	34.6183136
4000	36.8558366	34.9149037	34.6536452	34.6183136
5000	36.8558366	34.9149037	34.6536452	34.6183136
6000	36.8558366	34.9149037	34.6536452	34.6183136

Tabel 4.50: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 2) dan t = 5

Gamma (0.5, 1), t = 1, variansi secara analitik: 0.2128472222

N	A = 4	A = 5	A = 6	A = 7
1000	0.2158192	0.2131606	0.2128016	0.2127531
2000	0.2158192	0.2131606	0.2128016	0.2127531
3000	0.2158192	0.2131606	0.2128016	0.2127531
4000	0.2158192	0.2131606	0.2128016	0.2127531
5000	0.2158192	0.2131606	0.2128016	0.2127531
6000	0.2158192	0.2131606	0.2128016	0.2127531

Tabel 4.51: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 1) dan t = 1

Gamma (0.5, 1), t = 2, variansi secara analitik: 2.155555556

N	A = 4	A = 5	A = 6	A = 7
1000	2.2021408	2.161789	2.1563455	2.1556091
2000	2.2021408	2.161789	2.1563455	2.1556091
3000	2.2021408	2.161789	2.1563455	2.1556091
4000	2.2021408	2.161789	2.1563455	2.1556091
5000	2.2021408	2.161789	2.1563455	2.1556091
6000	2.2021408	2.161789	2.1563455	2.1556091

Tabel 4.52: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 1) dan t = 2

Gamma (0.5, 1), t = 3, variansi secara analitik: 9.140625

N	A = 4	A = 5	A = 6	A = 7
1000	9.418596	9.1780827	9.1456596	9.1412739
2000	9.418596	9.1780827	9.1456596	9.1412739
3000	9.418596	9.1780827	9.1456596	9.1412739
4000	9.418596	9.1780827	9.1456596	9.1412739
5000	9.418596	9.1780827	9.1456596	9.1412739
6000	9.418596	9.1780827	9.1456596	9.1412739

Tabel 4.53: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 1) dan t = 3

Gamma (0.5, 1), t = 4, variansi secara analitik: 27.02222222

N	A = 4	A = 5	A = 6	A = 7
1000	28.1091794	27.1687313	27.0420199	27.0248817
2000	28.1091794	27.1687313	27.0420199	27.0248817
3000	28.1091794	27.1687313	27.0420199	27.0248817
4000	28.1091794	27.1687313	27.0420199	27.0248817
5000	28.1091794	27.1687313	27.0420199	27.0248817
6000	28.1091794	27.1687313	27.0420199	27.0248817

Tabel 4.54: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 1) dan t = 4

Gamma (0.5, 1), t = 5, variansi secara analitik: 65.32118056

N	A = 4	A = 5	A = 6	A = 7
1000	68.6219651	65.7659758	65.3813303	65.3293084
2000	68.6219651	65.7659758	65.3813303	65.3293084
3000	68.6219651	65.7659758	65.3813303	65.3293084
4000	68.6219651	65.7659758	65.3813303	65.3293084
5000	68.6219651	65.7659758	65.3813303	65.3293084
6000	68.6219651	65.7659758	65.3813303	65.3293084

Tabel 4.55: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 1) dan t = 5

Gamma (0.5, 2), t = 1, variansi secara analitik: 0.0175568011

N	A = 4	A = 5	A = 6	A = 7
1000	0.0179003	0.0175592	0.0175131	0.0175069
2000	0.0179003	0.0175592	0.0175131	0.0175069
3000	0.0179003	0.0175592	0.0175131	0.0175069
4000	0.0179003	0.0175592	0.0175131	0.0175069
5000	0.0179003	0.0175592	0.0175131	0.0175069
6000	0.0179003	0.0175592	0.0175131	0.0175069

Tabel 4.56: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 2) dan t = 1

Gamma (0.5, 2), t = 2, variansi secara analitik: 0.2495246092

N	A = 4	A = 5	A = 6	A = 7
1000	0.2555336	0.2502823	0.2495737	0.2494779
2000	0.2555336	0.2502823	0.2495737	0.2494779
3000	0.2555336	0.2502823	0.2495737	0.2494779
4000	0.2555336	0.2502823	0.2495737	0.2494779
5000	0.2555336	0.2502823	0.2495737	0.2494779
6000	0.2555336	0.2502823	0.2495737	0.2494779

Tabel 4.57: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 2) dan t = 2

Gamma (0.5, 2), t = 3, variansi secara analitik: 1.17270147

N	A = 4	A = 5	A = 6	A = 7
1000	1.2070741	1.1772897	1.1732736	1.1727303
2000	1.2070741	1.1772897	1.1732736	1.1727303
3000	1.2070741	1.1772897	1.1732736	1.1727303
4000	1.2070741	1.1772897	1.1732736	1.1727303
5000	1.2070741	1.1772897	1.1732736	1.1727303
6000	1.2070741	1.1772897	1.1732736	1.1727303

Tabel 4.58: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 2) dan t = 3

Gamma (0.5, 2), t = 4, variansi secara analitik: 3.565322015

N	A = 4	A = 5	A = 6	A = 7
1000	3.6948737	3.5827499	3.5676402	3.5655964
2000	3.6948737	3.5827499	3.5676402	3.5655964
3000	3.6948737	3.5827499	3.5676402	3.5655964
4000	3.6948737	3.5827499	3.5676402	3.5655964
5000	3.6948737	3.5827499	3.5676402	3.5655964
6000	3.6948737	3.5827499	3.5676402	3.5655964

Tabel 4.59: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 2) dan t = 4

Gamma (0.5, 2), t = 5, variansi secara analitik: 8.60988276

N	A = 4	A = 5	A = 6	A = 7
1000	8.9942332	8.6616525	8.6168553	8.6107965
2000	8.9942332	8.6616525	8.6168553	8.6107965
3000	8.9942332	8.6616525	8.6168553	8.6107965
4000	8.9942332	8.6616525	8.6168553	8.6107965
5000	8.9942332	8.6616525	8.6168553	8.6107965
6000	8.9942332	8.6616525	8.6168553	8.6107965

Tabel 4.60: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 2) dan t = 5

Berikut adalah beberapa kasus yang tidak dapat dicari nilainya secara analitik.

Gamma (1, 0.5), t = 1, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	3.1697898	3.1180136	3.1110276	3.1100825
2000	3.1697898	3.1180136	3.1110276	3.1100825
3000	3.1697898	3.1180136	3.1110276	3.1100825
4000	3.1697898	3.1180136	3.1110276	3.1100825
5000	3.1697898	3.1180136	3.1110276	3.1100825
6000	3.1697898	3.1180136	3.1110276	3.1100825

Tabel 4.61: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 0.5) dan t = 1

Gamma (1, 0.5), t = 2, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	37.4696524	36.3509052	36.2001366	36.1797439
2000	37.4696524	36.3509052	36.2001366	36.1797439
3000	37.4696524	36.3509052	36.2001366	36.1797439
4000	37.4696524	36.3509052	36.2001366	36.1797439
5000	37.4696524	36.3509052	36.2001366	36.1797439
6000	37.4696524	36.3509052	36.2001366	36.1797439

Tabel 4.62: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 0.5) dan t = 2

Gamma (1, 0.5), t = 3, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	187.5023707	179.1733841	178.0517316	177.9000339
2000	187.5023707	179.1733841	178.0517316	177.9000339
3000	187.5023707	179.1733841	178.0517316	177.9000339
4000	187.5023707	179.1733841	178.0517316	177.9000339
5000	187.5023707	179.1733841	178.0517316	177.9000339
6000	187.5023707	179.1733841	178.0517316	177.9000339

Tabel 4.63: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 0.5) dan t = 3

Gamma (1, 0.5), t = 4, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	644.9506754	607.445071	602.3965174	601.7137684
2000	644.9506754	607.445071	602.3965174	601.7137684
3000	644.9506754	607.445071	602.3965174	601.7137684
4000	644.9506754	607.445071	602.3965174	601.7137684
5000	644.9506754	607.445071	602.3965174	601.7137684
6000	644.9506754	607.445071	602.3965174	601.7137684

Tabel 4.64: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 0.5) dan t = 4

Gamma (1, 0.5), t = 5, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	1780.0262883	1654.9155158	1638.0796513	1635.8029198
2000	1780.0262883	1654.9155158	1638.0796513	1635.8029197
3000	1780.0262883	1654.9155158	1638.0796513	1635.8029197
4000	1780.0262883	1654.9155158	1638.0796513	1635.8029197
5000	1780.0262883	1654.9155158	1638.0796513	1635.8029197
6000	1780.0262883	1654.9155158	1638.0796513	1635.8029197

Tabel 4.65: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 0.5) dan t = 5

Gamma (1, 2.5), t = 1, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	0.0260363	0.0254007	0.025315	0.0253034
2000	0.0260363	0.0254007	0.025315	0.0253034
3000	0.0260363	0.0254007	0.025315	0.0253034
4000	0.0260363	0.0254007	0.025315	0.0253034
5000	0.0260363	0.0254007	0.025315	0.0253034
6000	0.0260363	0.0254007	0.025315	0.0253034

Tabel 4.66: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 2.5) dan t = 1

Gamma (1, 2.5), t = 2, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	0.4409064	0.4292529	0.4276813	0.4274687
2000	0.4409064	0.4292529	0.4276813	0.4274687
3000	0.4409064	0.4292529	0.4276813	0.4274687
4000	0.4409064	0.4292529	0.4276813	0.4274687
5000	0.4409064	0.4292529	0.4276813	0.4274687
6000	0.4409064	0.4292529	0.4276813	0.4274687

Tabel 4.67: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 2.5) dan t = 2

Gamma (1, 2.5), t = 3, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	2.2708345	2.1958702	2.1857683	2.184402
2000	2.2708345	2.1958702	2.1857683	2.184402
3000	2.2708345	2.1958702	2.1857683	2.184402
4000	2.2708345	2.1958702	2.1857683	2.184402
5000	2.2708345	2.1958702	2.1857683	2.184402
6000	2.2708345	2.1958702	2.1857683	2.184402

Tabel 4.68: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 2.5) dan t = 3

Gamma (1, 2.5), t = 4, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	7.4435925	7.1347275	7.0931318	7.0875062
2000	7.4435925	7.1347275	7.0931318	7.0875062
3000	7.4435925	7.1347275	7.0931318	7.0875062
4000	7.4435925	7.1347275	7.0931318	7.0875062
5000	7.4435925	7.1347275	7.0931318	7.0875062
6000	7.4435925	7.1347275	7.0931318	7.0875062

Tabel 4.69: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 2.5) dan t = 4

Gamma (1, 2.5), t = 5, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	19.2594485	18.2808337	18.1491017	18.1312867
2000	19.2594485	18.2808337	18.1491017	18.1312867
3000	19.2594485	18.2808337	18.1491017	18.1312867
4000	19.2594485	18.2808337	18.1491017	18.1312867
5000	19.2594485	18.2808337	18.1491017	18.1312867
6000	19.2594485	18.2808337	18.1491017	18.1312867

Tabel 4.70: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (1, 2.5) dan t = 5

Gamma (0.5, 0.5), t = 1, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	1.4260961	1.4087823	1.406445	1.4061288
2000	1.4260961	1.4087823	1.406445	1.4061288
3000	1.4260961	1.4087823	1.406445	1.4061288
4000	1.4260961	1.4087823	1.406445	1.4061288
5000	1.4260961	1.4087823	1.406445	1.4061288
6000	1.4260961	1.4087823	1.406445	1.4061288

Tabel 4.71: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 0.5) dan t = 1

Gamma (0.5, 0.5), t = 2, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	14.1020862	13.8141359	13.7753009	13.7700476
2000	14.1020862	13.8141359	13.7753009	13.7700476
3000	14.1020862	13.8141359	13.7753009	13.7700476
4000	14.1020862	13.8141359	13.7753009	13.7700476
5000	14.1020862	13.8141359	13.7753009	13.7700476
6000	14.1020862	13.8141359	13.7753009	13.7700476

Tabel 4.72: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 0.5) dan t = 2

Gamma (0.5, 0.5), t = 3, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	61.6341417	59.8027003	59.5558677	59.5224813
2000	61.6341417	59.8027002	59.5558677	59.5224813
3000	61.6341417	59.8027002	59.5558677	59.5224813
4000	61.6341417	59.8027002	59.5558677	59.5224813
5000	61.6341417	59.8027002	59.5558677	59.5224813
6000	61.6341417	59.8027002	59.5558677	59.5224813

Tabel 4.73: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 0.5) dan t = 3

Gamma (0.5, 0.5), t = 4, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	190.0419627	182.6010284	181.5986572	181.463086
2000	190.0419627	182.6010284	181.5986572	181.463086
3000	190.0419627	182.6010284	181.5986572	181.463086
4000	190.0419627	182.6010284	181.5986572	181.463086
5000	190.0419627	182.6010284	181.5986572	181.463086
6000	190.0419627	182.6010284	181.5986572	181.463086

Tabel 4.74: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 0.5) dan t = 4

Gamma (0.5, 0.5), t = 5, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	479.389034	456.2858504	453.1746972	452.7539321
2000	479.389034	456.2858504	453.1746972	452.7539321
3000	479.389034	456.2858504	453.1746972	452.7539321
4000	479.389034	456.2858504	453.1746972	452.7539321
5000	479.389034	456.2858504	453.1746972	452.7539321
6000	479.389034	456.2858504	453.1746972	452.7539321

Tabel 4.75: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 0.5) dan t = 5

Gamma (0.5, 2.5), t = 1, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	0.0055894	0.0054329	0.0054118	0.0054089
2000	0.0055894	0.0054329	0.0054118	0.0054089
3000	0.0055894	0.0054329	0.0054118	0.0054089
4000	0.0055894	0.0054329	0.0054118	0.0054089
5000	0.0055894	0.0054329	0.0054118	0.0054089
6000	0.0055894	0.0054329	0.0054118	0.0054089

Tabel 4.76: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 2.5) dan t = 1

Gamma (0.5, 2.5), t = 2, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	0.1046911	0.1020583	0.1017031	0.101655
2000	0.1046911	0.1020583	0.1017031	0.101655
3000	0.1046911	0.1020583	0.1017031	0.101655
4000	0.1046911	0.1020583	0.1017031	0.101655
5000	0.1046911	0.1020583	0.1017031	0.101655
6000	0.1046911	0.1020583	0.1017031	0.101655

Tabel 4.77: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 2.5) dan t = 2

Gamma (0.5, 2.5), t = 3, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	0.5550478	0.5402058	0.5382044	0.5379337
2000	0.5550478	0.5402058	0.5382044	0.5379337
3000	0.5550478	0.5402058	0.5382044	0.5379337
4000	0.5550478	0.5402058	0.5382044	0.5379337
5000	0.5550478	0.5402058	0.5382044	0.5379337
6000	0.5550478	0.5402058	0.5382044	0.5379337

Tabel 4.78: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 2.5) dan t = 3

Gamma (0.5, 2.5), t = 4, variansi secara analitik: -

N	A = 4	A = 5	A = 6	A = 7
1000	1.7922203	1.7370864	1.7296561	1.7286511
2000	1.7922203	1.7370864	1.7296561	1.7286511
3000	1.7922203	1.7370864	1.7296561	1.7286511
4000	1.7922203	1.7370864	1.7296561	1.7286511
5000	1.7922203	1.7370864	1.7296561	1.7286511
6000	1.7922203	1.7370864	1.7296561	1.7286511

Tabel 4.79: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 2.5) dan t = 4

Gamma (0.5, 2.5), t = 5, variansi secara analitik: -

N	**A = 4**	**A = 5**	**A = 6**	**A = 7**
1000	4.4656175	4.3035412	4.2817095	4.2787568
2000	4.4656175	4.3035412	4.2817095	4.2787568
3000	4.4656175	4.3035412	4.2817095	4.2787568
4000	4.4656175	4.3035412	4.2817095	4.2787568
5000	4.4656175	4.3035412	4.2817095	4.2787568
6000	4.4656175	4.3035412	4.2817095	4.2787568

Tabel 4.80: Tabel Hasil Perhitungan Variansi Secara Numerik Untuk Gamma (0.5, 2.5) dan t = 5